THE MEASUREMENT OF 3-D RIGID BODY MOTION

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The effectiveness of restraint systems on the motion of human subjects cannot be completely evaluated without the description of 3-D motion of segments of the human subject, particularly that of the head.

Two techniques have been developed at HSRI to measure 3-D motion. One technique involves taking high-speed movies of two orthogonal views. The object being photographed is a fixture, rigidly mounted on the head. The inertial (x, y, z) coordinates of a minimum of three fixture-targets are obtained; then used to compute the 6-degrees of freedom necessary to describe the position in inertial space of the rigid body of interest (the head). This photometric technique has two major drawbacks: first, it is tedious, operator-dependent method, where each film must be analyzed frame-by-frame. Second, the accuracy of the method is acceptable for positions, but numerical differentiation results in highly magnified errors in velocities and accelerations.

The other technique to measure the 3-D motion uses accelerometers rigidly mounted on the rigid body being analyzed. The advantages of this method are obvious: automatic digitizing of accelerometer readings and numerical integration, which is a smoothing rather than error-magnifying operation.

Kinematic Equations

Let $(\hat{I}, \hat{J}, \hat{K})$ be an inertial (laboratory) cartesian reference frame, and $(\hat{i}, \hat{j}, \hat{k})$ be a cartesian frame, embedded in the moving rigid body, with origin at body-point C. There are two primary unknown vectors (6 scalar unknowns) to be measured: $\ddot{\vec{R}}$, the translational acceleration vector at the reference point C, and $\dot{\vec{\omega}}$, the angular acceleration vector of the rigid body. All other variables and unknowns can be derived from these 6 quantities.

Let $\ddot{\vec{r}}$ be the acceleration vector of an arbitrary body-point Q_1, measured experimentally with a triaxial accelerometer. The location Q_1 relative to C is given by a known (measured vector $\vec{\rho}_1$. Then with the rigidity assumption

$$\ddot{\vec{R}} + \dot{\vec{\omega}} \times \vec{\rho}_1 = \ddot{\vec{r}} - \vec{\omega} \times \vec{\omega} \times \vec{\rho}_1$$

The above vector equation may be considered as a system of 3 simultaneous equations in 6 unknowns. More equations are therefore needed to make the system a determinate one.

Instrumentation Approaches

It can be shown from rigid-body kinematics that a minimum of 3 points are necessary to locate, in 3-D space, a rigid body. Therefore, two additional points must be instrumented with accelerometers in order to obtain the necessary equations for a determinate system. However, only two configurations of accelerometers will result in such a system of equations. The 3-2-1 configuration [e.g., the Wayne-State approach] uses one triaxial, one biaxial and one uniaxial accelerometer at three points, Q_1, Q_2 and Q_3. The 2-2-2 configuration [e.g., the HSRI approach] uses 3 biaxial accelerometers at 3 points Q_1, Q_2 and Q_3.

The addition of any more accelerometers or rate-gyros make the system an over-determinate one. Redundant readings have been used in rigid-body motion measurement at CALSPAN and the Naval Aerospace Medical Research Laboratory.

Determinate Systems

The use of 6 accelerometers in the (3-2-1) or the (2-2-2) configuration is mathematically feasible and essentially boils down to solving two sets of equations: a set of three simultaneous, coupled, non-linear differential equations in $\dot{\omega}_1$, $\dot{\omega}_2$, $\dot{\omega}_3$; and a set of three algebraic equations in $\ddot{x}_c$, $\ddot{y}_c$, $\ddot{z}_c$.

The first set of differential equations may be abbreviated as:

$$\dot{w} = f(a,\omega)$$

where "a" are measured accelerations and "w" are computed angular velocities. This set of 3 equations, simple as they may seem, have been the center of discussion. It has been our experience, and that of others, that the numerical integration of these 3 simple equations usually accumulates enough errors to cause the most stable integration formula to exceed any reasonable accuracy. The cause of instability is not fully determined but probable sources are the instability of the Jacobian of the system under certain conditions, such as constant angular velocities, coupled with rounding-off errors in the numerical integration formula.

It was decided at HSRI that the use of 6 acceleration measurements, while it is mathematically feasible, is not fully reliable. Until such time when the reliability of a 6-accelerometer technique is proven, an alternate method is suggested.

Redundant Systems

The equations being integrated may be considered as a dynamic control system. The input (accelerometer readings) are added to angular velocity terms to produce angular accelerations which are integrated into an output (angular velocities). In order to counteract the instability problem, additional measurements may be made to be used to keep the output from unreasonable deviation. These redundant measurements may be obtained with linear accelerometers, angular accelerometers, or rate-gyros. The choice of linear accelerometers was dictated by the prohibitive cost and weight of angular accelerometers and rate gyros and the attractiveness of miniature linear accelerometers.

The system used at HSRI is a (3-3-3) configuration of 9 accelerometers. The difference between this system and the (3-2-2-2) system used at Wayne State is the number of mounting points. However, the character of the nine resulting equations is the same for both configurations.

With 9 equations and 6 unknowns, it is possible to select 6 equations which can be solved algebraically, thus eliminating altogether the necessity for numerical integration. These algebraic equations can be manipulated again into 2 sets of 3 equations. However, the set of 3 equations in the 3 angular velocities is a simultaneous, highly coupled, non-linear algebraic equation, for which there is no standard solving technique.

Such a technique has been developed at HSRI and may be characterized as a relaxation method. The elimination of any integration in this method makes it possible to obtain a point-by-point solution, i.e., a solution at a time point, independent from previous points.

This algebraic method has been tried on hypothetical motion and was successful where all other methods have failed. Experimental data has not been tried out because of hardware difficulties encountered during the experiments. It is felt, however, that this method will work for any experimental data, and has no limitations on the duration of the motion or the rate of digitizing.

The discussion of redundant systems must include the CALSPAN technique of using triaxial accelerometers. This method does not use directly the kinematic equations of motion in the solution; instead, the errors in these equations, resulting from errors in accelerometer readings, are minimized with respect to the 6 primary unknowns. This yields a different set of 6 equations which are solved (integration and algebraic solutions) to obtain the 3 linear and 3 angular accelerations. The method is elegant and simple; however, the stability of this method is not well established for impact motion.